Production of heavy states via relativistic bubble expansion

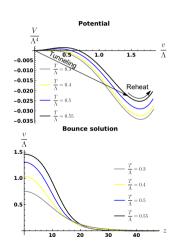
2010.02590 and 2101.05721 with Aleksandr Azatov and Wen Yin

Miguel Vanvlasselaer mvanvlas@sissa.it

SISSA

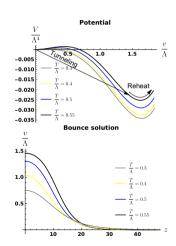
November 3, 2021

▶ Nucleation at critical radius $R_c \sim \frac{1}{T_n}$



- Nucleation at critical radius $R_c \sim \frac{1}{T_a}$
- ► Approximate solution of eq. of motion

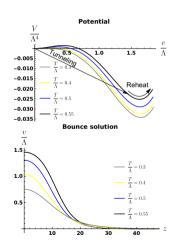
$$\phi(r) = rac{v}{2}igg(1 - anhigg(rac{r - R_c}{L_w}igg)igg), \quad L_w \sim rac{1}{M}$$



- Nucleation at critical radius $R_c \sim \frac{1}{T_a}$
- ► Approximate solution of eq. of motion

$$\phi(r) = rac{v}{2}igg(1 - anhigg(rac{r-R_c}{L_w}igg)igg), \quad L_w \sim rac{1}{M}$$

ightharpoonup Energy released $\epsilon \equiv \Delta V$

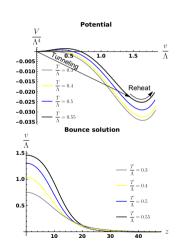


- Nucleation at critical radius $R_c \sim \frac{1}{T_o}$
- ► Approximate solution of eq. of motion

$$\phi(r) = rac{v}{2}igg(1 - anhigg(rac{r-R_c}{L_w}igg)igg), \quad L_w \sim rac{1}{M}$$

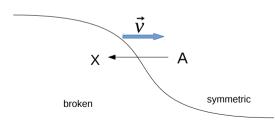
- ightharpoonup Energy released $\epsilon \equiv \Delta V$
- ► Pressureless expansion

$$\gamma_{wp} \equiv \frac{1}{\sqrt{1 - v_w^2}} \quad \Rightarrow \gamma_{wp} \propto R$$



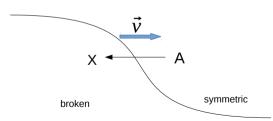
Velocity

- ► Terminal velocity regime: $\Delta V = \Delta \mathcal{P}(\gamma_{wp}^{MAX})$ \Rightarrow GW signal from sound waves
- ▶ Runaway regime: $\Delta V > \Delta \mathcal{P}(\gamma_{wp}^{MAX})$ $\Rightarrow \gamma_{wp} \propto R$ GW signal from bubble collision
- Generic method: solve the full coupled system of Boltzmann equations



Velocity

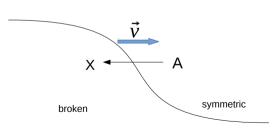
- ► Terminal velocity regime: $\Delta V = \Delta \mathcal{P}(\gamma_{wp}^{MAX})$ \Rightarrow GW signal from sound waves
- ► Runaway regime: $\Delta V > \Delta \mathcal{P}(\gamma_{wp}^{MAX})$ $\Rightarrow \gamma_{wp} \propto R$ GW signal from bubble collision
- Generic method: solve the full coupled system of Boltzmann equations
- ▶ Regime $\gamma \gg 1$ $\mathcal{C} \rightarrow 0$: Collisionless system of quasi-particles



Velocity

- ► Terminal velocity regime: $\Delta V = \Delta \mathcal{P}(\gamma_{wp}^{MAX})$ \Rightarrow GW signal from sound waves
- ▶ Runaway regime: $\Delta V > \Delta \mathcal{P}(\gamma_{wp}^{MAX})$ $\Rightarrow \gamma_{wp} \propto R$ GW signal from bubble collision
- Generic method: solve the full coupled system of Boltzmann equations
- ▶ Regime $\gamma \gg 1$ $\mathcal{C} \rightarrow 0$: Collisionless system of quasi-particles
- ▶ Pressure on the wall [2010.02590]

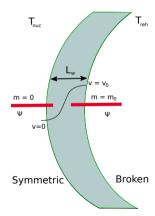
$$\mathcal{P} = \int rac{p_z d^3 p}{p_0 (2\pi)^3} f_A(p) imes \sum_X \int dP_{A o X} (p_A^Z - p_X^Z)$$



Pressure from 1 to 1

Pressure on the wall

$$\mathcal{P} = \int rac{p_z d^3 p}{p_0 (2\pi)^3} f_{\Psi_i}(p) imes \sum_{\Psi_i} \int rac{dP_{i
ightarrow i}(p_{i,out}^Z - p_{i,in}^Z)}{p_i^2}$$



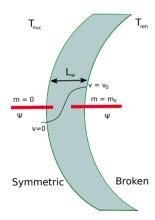
Pressure from 1 to 1

Pressure on the wall

$$\mathcal{P} = \int rac{p_z d^3 p}{p_0 (2\pi)^3} f_{\Psi_i}(p) imes \sum_{\Psi_i} \int rac{d P_{i
ightarrow i}(p_{i,out}^Z - p_{i,in}^Z)}{p_i^Z}$$

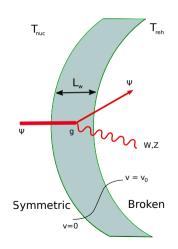
► LO relativistic pressure [0903.4099]

$$\int dP_{i o i} o 1, \qquad (p_{i,out}^Z - p_{i,in}^Z) pprox rac{\Delta m_i^2}{2E}$$
 $\Rightarrow \mathcal{P}_{1 o 1} o \sum_i rac{\Delta m_i^2 T^2}{24} c_i,$
 $\Delta m_i^2 \equiv m_{bro,i}^2 - m_{sym,i}^2$



▶ NLO relativistic pressure; gauge bosons *V*[1703.08215]

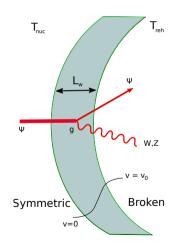
$$E_{\psi} \sim p_{z,\psi} \sim \gamma_{wp} \, T \gg T$$
 WKB applicable $rac{dp_z}{dz} \ll p_z^2$



▶ NLO relativistic pressure; gauge bosons V[1703.08215]

$$E_{\psi} \sim
ho_{z,\psi} \sim \gamma_{wp} \, T \gg T$$
 WKB applicable $rac{d
ho_z}{dz} \ll
ho_z^2$

$$\int dP_{\Psi o V\Psi} \sim egin{cases} 0 & ext{if TS} \ rac{g^3 v}{16\pi^2} & ext{Wall breaks TS} \end{cases}$$

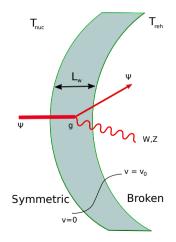


▶ NLO relativistic pressure; gauge bosons V[1703.08215]

$$E_{\psi} \sim
ho_{z,\psi} \sim \gamma_{wp} \, T \gg T$$
 WKB applicable $rac{d
ho_z}{dz} \ll
ho_z^2$

$$\int dP_{\Psi o V\Psi} \sim egin{cases} 0 & ext{if TS} \ rac{g^3 v}{16\pi^2} & ext{Wall breaks TS} \end{cases}$$

▶ Soft bosons emission $\Delta p \sim m_V$



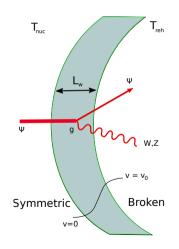
▶ NLO relativistic pressure; gauge bosons V[1703.08215]

$$E_{\psi} \sim p_{z,\psi} \sim \gamma_{wp} T \gg T$$
 WKB applicable $\frac{dp_z}{dz} \ll p_z^2$

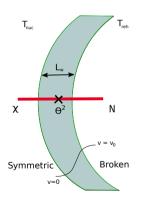
$$\int dP_{\Psi \to V\Psi} \sim \begin{cases} 0 & \text{if TS} \\ \frac{g^3 v}{16\pi^2} & \text{Wall breaks TS} \end{cases}$$

- ▶ Soft bosons emission $\Delta p \sim m_V$
- Pressure induced

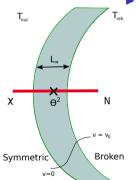
$$\Rightarrow \boxed{\mathcal{P}_{1 o 2} \sim \sum_{i} g_{i} rac{g^{3} v}{16\pi^{2}} \gamma_{wp} T^{3}}$$



ightharpoonup Q; what about $M_N\gg T_{nuc}$? Transition is dictated by fields $m\lesssim T_{nuc}$

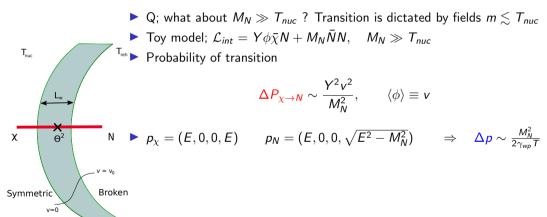


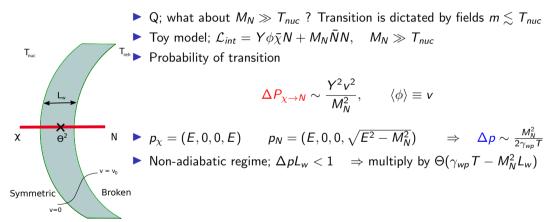
- ightharpoonup Q; what about $M_N\gg T_{nuc}$? Transition is dictated by fields $m\lesssim T_{nuc}$
- ► Toy model; $\mathcal{L}_{int} = Y \phi \bar{\chi} N + M_N \bar{N} N$, $M_N \gg T_{nuc}$

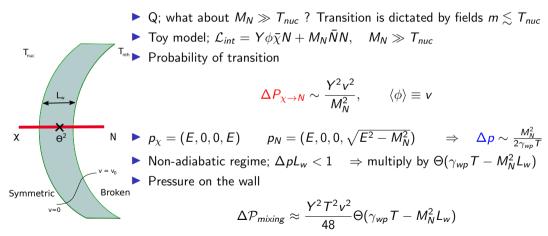


- ightharpoonup Q; what about $M_N\gg T_{nuc}$? Transition is dictated by fields $m\lesssim T_{nuc}$
- ► Toy model; $\mathcal{L}_{int} = Y \phi \bar{\chi} N + M_N \bar{N} N$, $M_N \gg T_{nuc}$
- T_{reh} Probability of transition Broken Symmetric

$$\Delta P_{\chi o N} \sim rac{Y^2 v^2}{M_N^2}, \qquad \langle \phi
angle \equiv v$$



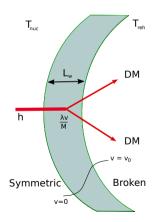




Pressure depends on M_N only in the Θ -function

Production mechanism[2101.05721]

lacktriangle Portal Dark Matter: $\mathcal{L} \supset -rac{\lambda}{2}h^2\phi^2 - M_\phi^2\phi^2 - V(h)$

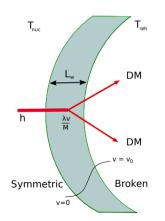


Production mechanism[2101.05721]

- ▶ Portal Dark Matter: $\mathcal{L} \supset -\frac{\lambda}{2}h^2\phi^2 M_\phi^2\phi^2 V(h)$
- Non-vanishing VEV:

$$h o h + v$$
 trilinear coupling $\mathcal{L} \supset -\lambda v h \phi^2$

$$\mathcal{L}\supset -\lambda \mathit{vh}\phi^2$$



Production mechanism 2101.05721

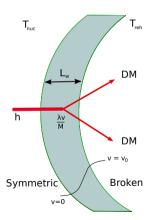
- ▶ Portal Dark Matter: $\mathcal{L} \supset -\frac{\lambda}{2}h^2\phi^2 M_\phi^2\phi^2 V(h)$
- Non-vanishing VEV:

 $h \to h + v$ trilinear coupling $\mathcal{L} \supset -\lambda v h \phi^2$

$$\mathcal{L}\supset -\lambda v h \phi^2$$

▶ In the wall frame: $E_h \sim p_{z,h} \sim \gamma_{wp} T \gg T$

$$P_{h o \phi \phi} \sim rac{\lambda^2 v^2}{M_\phi^2} \Theta(\gamma_{wp} T - M_\phi^2 L_w)$$



Production mechanism[2101.05721]

- ▶ Portal Dark Matter: $\mathcal{L} \supset -\frac{\lambda}{2}h^2\phi^2 M_\phi^2\phi^2 V(h)$
- ► Non-vanishing VEV:

$$h o h + v$$
 trilinear coupling $\mathcal{L} \supset -\lambda v h \phi^2$

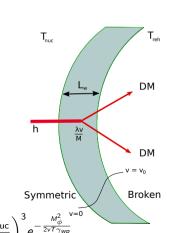
▶ In the wall frame: $E_h \sim p_{z,h} \sim \gamma_{wp} T \gg T$

$$P_{h
ightarrow\phi\phi}\simrac{\lambda^2 v^2}{M_\phi^2}\Theta(\gamma_{wp}T-M_\phi^2L_w)$$

 \triangleright Behind the wall, accumulation of relics ϕ

$$n_{\phi}^{\mathsf{BE}} pprox rac{T^3}{12\pi^4} rac{\lambda^2 v^2}{M_{\phi}^2} e^{-rac{M_{\phi}^2}{2vT\gamma_{wp}}} + \mathcal{O}(1/\gamma_w)$$

$$\Rightarrow \Omega_{\phi, \mathsf{BE}}^{\mathsf{today}} h^2 \approx 5.4 \times 10^5 \times \left(\frac{1}{g_{\star \mathcal{S}}(T_{\mathsf{reh}})}\right) \left(\frac{\lambda^2 v}{M_{\phi}}\right) \left(\frac{v}{\mathsf{GeV}}\right) \left(\frac{T_{\mathsf{nuc}}}{T_{\mathsf{reh}}}\right)^3 e^{-\frac{M_{\phi}^2}{2 v T_{\mathsf{Twp}}}}$$



▶ No wall: transition $\chi \to N$ forbidden by four-momentum conservation

$$p_{\chi} = (E, 0, 0, E)$$
 $p_{N} = (E, 0, 0, \sqrt{E^{2} - M^{2}})$

▶ No wall: transition $\chi \to N$ forbidden by four-momentum conservation

$$p_{\chi} = (E, 0, 0, E)$$
 $p_{N} = (E, 0, 0, \sqrt{E^{2} - M^{2}})$

lacktriangle with wall: z-momentum conservation broken $\Delta p_z
eq 0$



transition $\chi \to N$ allowed if enough energy in wall frame \Rightarrow Pressure on the wall

▶ No wall: transition $\chi \to N$ forbidden by four-momentum conservation

$$p_{\chi} = (E, 0, 0, E)$$
 $p_{N} = (E, 0, 0, \sqrt{E^{2} - M^{2}})$

lacktriangle with wall: z-momentum conservation broken $\Delta p_z
eq 0$



transition $\chi \to N$ allowed if enough energy in wall frame \Rightarrow Pressure on the wall

▶ Generalisation to $1 \rightarrow 2$: interesting for generation of DM

lacktriangle No wall: transition $\chi \to N$ forbidden by four-momentum conservation

$$p_{\chi} = (E, 0, 0, E)$$
 $p_{N} = (E, 0, 0, \sqrt{E^{2} - M^{2}})$

lacktriangle with wall: z-momentum conservation broken $\Delta p_z
eq 0$



transition $\chi \to N$ allowed if enough energy in wall frame \Rightarrow Pressure on the wall

- ▶ Generalisation to $1 \rightarrow 2$: interesting for generation of DM
- ▶ Caveat: wall suppression for $\Delta p_z L_w > 1$

Back up

Back up slides

Can γ_{wp} be large enough to produce ϕ of M_{ϕ} ?

Transition strong enough : $|\Delta V > \Delta P_{LO}|$

$$\Delta V > \Delta \mathcal{P}_{LO}$$

Transition sector without Gauge Bosons

$$\Delta \mathcal{P} = \Delta \mathcal{P}_{LO}$$



Runaway regime: acceleration until collision

$$\gamma_{w, \text{MAX}} pprox rac{M_{
m pl} T_{
m nuc}}{v^2}$$

$$\Rightarrow \qquad \left| M_{\phi}^{\mathrm{MAX}} \sim T_{\mathrm{nuc}} \left(rac{M_{\mathrm{pl}}}{v}
ight)^{1/2}
ight|$$

Can γ_{wn} be large enough to produce ϕ of M_{ϕ} ?

Transition strong enough : $|\Delta V > \Delta P_{LO}|$

$$\Delta V > \Delta \mathcal{P}_{LO}$$

Transition sector without Gauge Bosons

$$\Delta \mathcal{P} = \Delta \mathcal{P}_{LO}$$

Transition sector with Gauge Bosons

$$\Delta \mathcal{P} = \Delta \mathcal{P}_{LO} + \Delta \mathcal{P}_{NLO}$$

Runaway regime: acceleration until collision

$$\Downarrow$$

$$\gamma_{w,\text{MAX}} \approx \frac{M_{\text{pl}} T_{\text{nuc}}}{v^2}$$

$$ho M_{\phi}^{
m MAX} \sim T_{
m nuc} \left(rac{M_{
m pl}}{V}
ight)^{1/2}$$

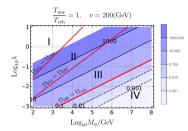
$$\gamma_{w,\mathsf{MAX}} pprox \mathsf{Min}igg[rac{M_{\mathsf{pl}}\,T_{\mathsf{nuc}}}{v^2}, rac{16\pi^2}{g_ig_{\mathsf{gauge}}^3}igg(rac{v}{T_{\mathsf{nuc}}}igg)^3igg]$$

$$\Rightarrow \boxed{M_{\phi}^{\mathsf{MAX}} \sim \mathrm{Min} \bigg[T_{\mathsf{nuc}} \bigg(\frac{M_{\mathsf{pl}}}{v} \bigg)^{1/2}, 4\pi v \bigg(\frac{v}{T_{\mathsf{nuc}}} \bigg) \bigg]}$$

Putting ourselves in our universe: $\Omega_{\phi, {\sf BE}}^{\sf today} h^2 pprox 0.12$ and freeze-out

▶ If ϕ was in thermal equilibrium:

$$\Omega_{\phi,\; \mathsf{FO}}^{\mathsf{today}} h^2 pprox 0.1 igg(rac{0.03}{\lambda}igg)^2 igg(rac{M_\phi}{100\; \mathsf{GeV}}igg)^2$$



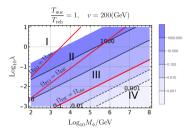
Large over-production of DM

Remedy to the over-production?

Putting ourselves in our universe: $\Omega_{\phi,\mathsf{BE}}^{\mathsf{today}} h^2 \approx 0.12$ and freeze-out

▶ If ϕ was in thermal equilibrium:

$$\Omega_{\phi,\; \mathsf{FO}}^{\mathsf{today}} h^2 pprox 0.1 igg(rac{0.03}{\lambda}igg)^2 igg(rac{M_\phi}{100\; \mathsf{GeV}}igg)^2$$



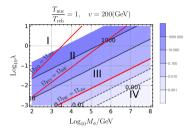
Large over-production of DM

Remedy to the over-production?

Putting ourselves in our universe: $\Omega_{\phi,\mathsf{BE}}^{\mathsf{today}} h^2 pprox 0.12$ and freeze-out

▶ If ϕ was in thermal equilibrium:

$$\Omega_{\phi,\;\mathsf{FO}}^{\mathsf{today}} \, h^2 pprox 0.1 igg(rac{0.03}{\lambda} igg)^2 igg(rac{M_\phi}{100\;\mathsf{GeV}} igg)^2$$



Large over-production of DM

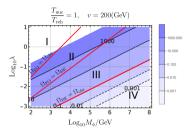
Remedy to the over-production?

► Annihilation of DM after production: $\Gamma_{ann} \sim \sigma_{\phi\phi} v_{rel} n_{\phi} > H \Rightarrow$ Annihilation

Putting ourselves in our universe: $\Omega_{\phi,\mathsf{BE}}^{\mathsf{today}} h^2 \approx 0.12$ and freeze-out

▶ If ϕ was in thermal equilibrium:

$$\Omega_{\phi,\; \mathsf{FO}}^{\mathsf{today}} h^2 pprox 0.1 igg(rac{0.03}{\lambda}igg)^2 igg(rac{M_\phi}{100\; \mathsf{GeV}}igg)^2$$



Large over-production of DM

Remedy to the over-production?

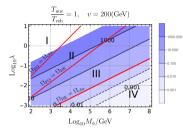
- lacktriangle Annihilation of DM after production: $\Gamma_{\sf ann} \sim \sigma_{\phi\phi} v_{
 m rel} n_{\phi} > H \Rightarrow {\sf Annihilation}$
- ▶ Large supercooling: $\left(\frac{T_{\text{nuc}}}{T_{\text{reh}}}\right) \ll 1$

Putting ourselves in our universe: $\Omega_{\phi,\mathsf{BE}}^{\mathsf{today}} h^2 pprox 0.12$ and freeze-out

▶ If ϕ was in thermal equilibrium:

$$\Omega_{\phi,\;\mathsf{FO}}^{\mathsf{today}} h^2 pprox 0.1 igg(rac{0.03}{\lambda}igg)^2 igg(rac{M_\phi}{100\;\mathsf{GeV}}igg)^2$$

$$\qquad \qquad \Omega_{\phi, \rm BE}^{\rm today} h^2 \approx 5.4 \times 10^5 \bigg(\frac{1}{g_{\star S}(T_{\rm reh})} \bigg) \bigg(\frac{\lambda^2 v}{M_{\phi}} \bigg) \bigg(\frac{v}{\rm GeV} \bigg) \bigg(\frac{T_{\rm nuc}}{T_{\rm reh}} \bigg)^3$$

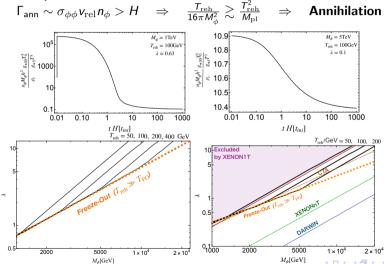


Large over-production of DM

Remedy to the over-production?

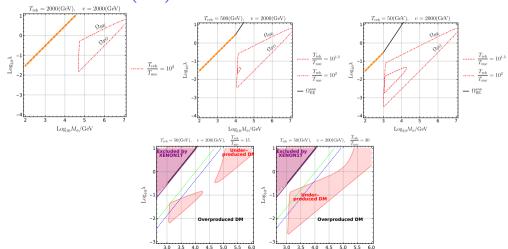
- ▶ Annihilation of DM after production: $\Gamma_{\sf ann} \sim \sigma_{\phi\phi} v_{\rm rel} n_{\phi} > H \Rightarrow {\sf Annihilation}$
- ▶ Large supercooling: $\left(\frac{T_{\text{nuc}}}{T_{\text{reh}}}\right) \ll 1$
- $ightharpoonup \phi$ was never in thermal equilibrium

Annihilation after production



Conclusion

Large supercooling: $\left(\frac{T_{\text{nuc}}}{T_{\text{reh}}}\right) \ll 1$



 $\text{Log}_{10}M_{\phi}/\text{GeV}$

 $\text{Log}_{10}M_{\phi}/\text{GeV}$

ϕ was never in thermal equilibrium

▶ Reheating after inflation was too late:

$$T_R \ll T_{\sf FO} pprox rac{M_\phi}{20}$$
 (No FO condition)

ϕ was never in thermal equilibrium

Reheating after inflation was too late:

$$T_R \ll T_{ extsf{FO}} pprox rac{M_\phi}{20}$$
 (No FO condition)

In our universe

$$\Omega_{\phi,\mathsf{BE}}^{\mathsf{today}} h^2 = 0.12 \quad \Rightarrow \quad M_\phi pprox 5 imes 10^4 \lambda^2 igg(rac{v}{\mathsf{GeV}}igg)^2 \; \mathsf{GeV}$$

ϕ was never in thermal equilibrium

Reheating after inflation was too late:

$$T_R \ll T_{ extsf{FO}} pprox rac{M_\phi}{20}$$
 (No FO condition)

► In our universe

$$\Omega_{\phi,\,\mathrm{BE}}^{\mathrm{today}} h^2 = 0.12 \quad \Rightarrow \quad M_{\phi} pprox 5 imes 10^4 \lambda^2 igg(rac{v}{\mathrm{GeV}}igg)^2 \; \mathrm{GeV}$$

Maximal DM mass that can be produced in that way:

$$\lambda < 4\pi \quad \Rightarrow M_{\phi} < M_{\phi}^{\mathsf{MAX}} pprox 5 imes 10^6 igg(rac{v}{\mathsf{GeV}}igg)^2 \; \mathsf{GeV}$$

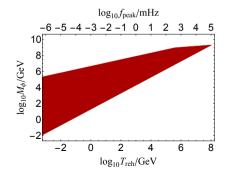
observable GW from DM production mechanism

Strong GW signal

Strong GW signal obtained for: long supercooling, large bubbles, fast walls

 \Rightarrow This is exactly the regime for optimal DM production

$$10^{-6}~{
m mHz} \lesssim f_{
m peak} \lesssim 100~{
m Hz}$$



observable GW from DM production mechanism

Strong GW signal

Strong GW signal obtained for: long supercooling, large bubbles, fast walls

⇒ This is exactly the regime for optimal DM production

$$10^{-6} \text{ mHz} \lesssim f_{\text{peak}} \lesssim 100 \text{ Hz}$$

$$\log_{10} f_{\text{peak}} / \text{mHz}$$

$$-6 - 5 - 4 - 3 - 2 - 1 \text{ 0 1 2 3 4 5}$$

$$10 \\ 8 \\ 6 \\ 6 \\ 70 \\ 9 \\ 9 \\ 100 \\ -2 \\ 10g_{10} T_{\text{rch}} / \text{GeV}$$

